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Bank-to-Turn Missile Autopilot Design Via Observer-Based Command Governor Approach

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I. Introduction

IN recent years there have been substantial theoretical advancements in the field of feedback control of dynamic systems under input and/or state-related constraints.[1] In this connection, two main directions have emerged. On one side, nonlinear control design methodologies explicitly take into account constraints in the design phase and result in control strategies that ensure, within given conditions, closed-loop stability, fulfillment of constraints, and other control specifications. On the other side, control techniques finalized to modify online the command provided by a nominal linear controller so as to possibly prevent or handle constraint violations. Antiwindup and command governor (CG) approaches are examples of this second class of techniques. Within the first, problems of semiglobal or global stabilization of linear systems under input saturation have been addressed, for example, in Ref. 2 via a gain-scheduling algebraic Riccati equation approach. The theory of positive invariant sets has been extensively used in solving regulation and tracking problems in the presence of constraints.[3] Other contributions come from predictive control methodology,[4] which has been used to tackle tracking problems under constraints of various types.

A CG is a nonlinear device that is added to a primal compensated control system, as depicted in Fig. 1. The latter, in the absence of the CG, is designed so as to perform satisfactorily in the absence of constraint violations (typically in small-signal regimes). Whenever necessary, the CG modifies the input to the primal control system so as to possibly avoid violation of constraints. In this way, a system equipped with a CG takes a special simplified structure at the cost typically of performance degradation with respect to more general approaches, for example, constrained predictive control. CG usage can be however justified in applications wherein a massive amount of flops per sampling time is not allowed, and/or one is typically only commissioned to add to existing standard proportional–integrative–derivative (PID)-like compensators peripheral units, which, as CGs, do not change the primal compensated control system. Specific merits of the CG approach in dealing with constraints are that it can handle absolute and incremental constraints on input and state-related variables of the plant and that the numerical burdens of the online computation can be modulated according to the available computing power, ranging from solving online convex multidimensional optimization problems to consulting look-up tables.

Typically, at each sampling time the CG action computation consists of solving a constrained quadratic-programming problem, which depends on the current desired reference, state, and prescribed constraints. A standard QP solver can be used for such a task, and the CG algorithm can be implemented on any off-the-shelf digital-signal-processing board. In fact, the computational power required for obtaining the CG action computation can be modulated by an appropriate CG design, and the current microprocessor technology allows the achievement of typical sampling rates used in aerospace applications.

Studies on CG designed along these lines have already appeared in Ref. 5. For CGs approached from different perspectives, see Refs. 6 and 7. All of the preceding works are concerned with the complete state information case, that is, all components of the state are available to the CG. However, this assumption is not usually satisfied in aerospace applications because of space and load limitations. Here such a restriction is removed, as only a noisy partial state information is assumed to be available to the CG.

Finally, the Note discusses an application of the proposed CG approach to a bank-to-turn missile autopilot design. Specifically, the missile is required to track the demanded pitch acceleration as closely as possible while maintaining the elevator angle between the given bounds. Because the pitch-angle dynamic model has unstable modes, avoiding constraint violation is of mandatory importance in order to maintain stability. Prior works on applying AW and CG schemes to bank-to-turn missile control problems include Refs. 8 and 9. Here we have used the same model and PI controller used in Ref. 8, where the CG approach of Ref. 6 was used, so that comparisons are possible by contrasting the respective responses. The present approach results achieve larger constrained stability regions than that achieved in Ref. 8.

Throughout the Note, vectors and matrices will be denoted respectively by lower-case (x as an example) and upper-case (X) standard letter styles, whereas the calligraphic one (χ) will be used to denote sets.

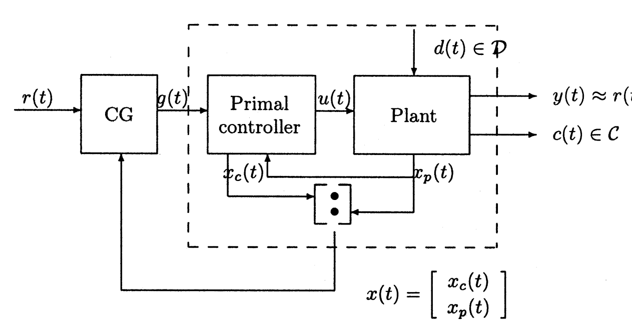

Fig. 1 Command governor structure.

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II. Problem Formulation and Main Results

Consider the following linear time-invariant system

$$
\begin{aligned}
x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t), \qquad x(0) \in \chi(0)\\
y(t) &= H_y x(t)\\
z(t) &= H_z x(t) + \xi(t)\\
c(t) &= H_c x(t) + Lg(t) + \tilde{L} d(t)
\end{aligned} \tag{1}
$$

where $t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$; $x(t) \in \mathbb{R}^n$ is the state; $g(t) \in \mathbb{R}^m$ the manipulable command input, which, if no constraints were present, would coincide with the output reference $r(t) \in \mathbb{R}^m$; $d(t) \in \mathbb{R}^n$ an exogenous plant disturbance; $y(t) \in \mathbb{R}^m$ the output, which is required to track the reference signal $r(t)$; $z(t) \in \mathbb{R}^p$ the measured output; $\xi(t) \in \mathbb{R}^p$ a measurement noise; $c(t) \in \mathbb{R}^{n_c}$ the constrained vector; and $\chi(0)$ an a priori given set accounting for the existing initial state uncertainty.

It is assumed that

$$
\left.\begin{array}{l}\text{The system (1) is asymptotically stable and offset-free,}\\ \text{i.e., } H_y(I - \Phi)^{-1} G = I_m\end{array}\right\} \tag{2}
$$

One important instance of Eq. (1) consists of a linear plant under stabilizing feedback control. In this respect, the plant could have been compensated so as to satisfy stability and performance requirements without enforcing the prescribed constraints.

The plant disturbance $d(t)$ and the measurement noise $\xi(t)$ are assumed to belong to given sets $\mathcal{D} \subset \mathbb{R}^n$ and, respectively, $\Xi \subset \mathbb{R}^p$, namely,

$$
d(t) \in \mathcal{D}, \qquad \xi(t) \in \Xi, \qquad \forall t \in \mathbb{Z}_+ \tag{3}
$$

The constrained vector $c(t)$ is required to fulfill the pointwise-in-time set-membership constraint

$$
c(t) \in \mathcal{C}, \qquad \forall t \in \mathbb{Z}_+ \tag{4}
$$

with $\mathcal{C} \subset \mathbb{R}^{n_c}$, a prescribed constraint set. It is further assumed that

$$
\left.\begin{array}{l}\text{1) } \mathcal{D} \text{ and } \Xi \text{ are compact and convex sets with } 0_n \in \mathcal{D} \text{ and } 0_p \in \Xi\\ \text{2)}\\ \mathcal{C} \text{ and } \chi(0) \text{ are compact and convex sets with nonempty interiors}\end{array}\right\} \tag{5}
$$

The exact way to characterize the evolutions of all states compatible with measurements and exogenous disturbance bounds is that of introducing the following set-valued recursions:

$$
\chi(t \mid t) := \chi(t \mid t-1) \bigcap \chi_z(t), \qquad \chi(0 \mid -1) = \chi(0) \tag{6}
$$

$$
\chi(t+1 \mid t) := \Phi \chi(t \mid t) \oplus \{Gg(t)\} \oplus G_d \mathcal{D} \tag{7}
$$

where $\chi(t_2 \mid t_1)$ denotes the set of all states of Eq. (1), which, at time t_2, are compatible with measurements collected up to time t_1, $t_2 \geq t_1$, and

$$
\chi_z(t) := \left\{x \in \mathbb{R}^n : z(t) - H_z x \in \Xi\right\} \tag{8}
$$

represents the set of states compatible with measurement $z(t)$. In Eq. (7) and hereafter the sum involving sets has to be intended in the Minkowsky vector sum sense, namely, $\mathcal{U} \oplus \mathcal{V} := \{u + v : u \in \mathcal{U}, v \in \mathcal{V}\}$.

Then, the problem is to design a memory-less device,

$$
g(t) := \underset{\sim}{g}(\chi(t \mid t), r(t)) \tag{9}
$$

in such a way that, under suitable conditions and for all possible disturbance sequences $d(\,)$ and $\xi(\,)$, constraints (4) are fulfilled, and possibly $y(t) \approx r(t)$. Notice that the state $x(t)$ is inaccessible, as the only measurable variable is $z(t)$, and the CG action depends on the whole set $\chi(t \mid t)$.

The main difficulty in extending the basic state-based CG schems to the output case is how to take care of the estimation errors in ensuring *feasibility retention*, a property that ensures that solvability of Eq. (9) at time instant 0 implies its solvability at every future time instants $t \in \mathbb{Z}_+$. In Ref. 10 output-based CG shemes based on the exact and the approximated determinations of $\chi(t \mid t)$ were presented. However, the exact approach leads to CG schemes that, although performing in a no-conservative way and enjoying feasibility retention, can suffer no constant computational burdens per step. This happens because the numerical complexity required for exactly characterizing $\chi(t \mid t)$ can grow without limit as new measurements are collected. On the other hand, approaches based on fixed-shaped outer approximations of $\chi(t \mid t)$, such as ellipsoids or polytopes that are not affected by this numerical problem, suffer from theoretical difficulties in ensuring any feasibility retention property, unless by accepting large performance degradations (see Ref. 10 for details).

Hereafter, a simpler approach is presented consisting of using a standard Luenberger observer and a fixed outer approximation of the state estimation error. Specifically, we consider CG schemes as follows:

$$
g(t) := \underset{\sim}{g}(\hat{x}(t) + \tilde{\chi}, \tau(t)) \tag{10}
$$

where the estimate $\hat{x}(t)$ is provided by a linear state observer and $\tilde{\chi}$ is a convenient compact and convex set such that $\hat{x}(t) + \tilde{\chi} \supset \chi(t \mid t), \forall t \in \mathbb{Z}_+$.

An interesting feature of the present CG approach is that one can synthesize the observer without taking into account constraints. Moreover, the latter need not share with the plant all measurement data available for control. Although this feature can appear quite artificial in practical applications, it can be of help in some situations. For example, one case when the CG is added to a pre-existent well-tuned closed-loop system and for some reason one or more plant outputs is not directly available to the CG, or it is so noisy that is preferable not using it for avoiding conservative results. Because many different situations can arise, hereafter we assume that a suitable Luenberger observer is given. Then, the overall system becomes

$$
\begin{aligned}
x(t+1) &= \Phi x(t) + Gg(t) + G_d d(t), \qquad x(0) \in \chi(0)\\
\hat{x}(t+1) &= \Phi \hat{x}(t) + Gg(t) + K[z(t) - \hat{z}(t)], \qquad \hat{x}(0) = 0_n\\
y(t) &= H_y x(t)\\
z(t) &= H_z x(t) + \xi(t)\\
\hat{z}(t) &= H_z \hat{x}(t)\\
c(t) &= H_c x(t) + Lg(t) + \tilde{L} d(t)
\end{aligned} \tag{11}
$$

Now $\hat{x}(t)$ and $c(t)$ in Eq. (11) can be rewritten in terms of the estimation error $\tilde{x}(t) := x(t) - \hat{x}(t)$ as

$$
\begin{aligned}
\tilde{x}(t+1) &= (\Phi + K H_z)\tilde{x}(t) + G_d d(t) + K\xi(t)\\
\hat{x}(t+1) &= \Phi \hat{x}(t) + Gg(t) + K H_z \tilde{x}(t) - K\xi\\
c(t) &= H_c \hat{x}(t) + H_c \tilde{x}(t) + Lg(t) + \tilde{L} d(t)
\end{aligned} \tag{12}
$$

Because $\Phi_0 := \Phi + K H_z$ is a stable matrix by design, it is possible to compute the worst-case estimation error set

$$
\mathcal{R}^*(\chi(0)) = \bigcup_{t=0}^{\infty} \mathcal{R}_i^*(\chi(0))
$$

$$
\mathcal{R}_t^*(\chi(0)) := \left\{\Phi_0^t \chi(0) \oplus \bigoplus_{i=0}^{t-1} \left(\Phi_0^i [G_d \mathcal{D} \oplus K\,\Xi]\right)\right\} \tag{13}
$$

which represents the bounded set of all possible estimation errors. Next, we assume that a suitable convex outer approximation

$$
\tilde{\chi} \supset \mathcal{R}^*(\chi(0)) \tag{14}
$$

can be computed. Then, one can treat the observation error as a disturbance $\bar{x}(t) \in \bar{\chi}, \forall t \in \mathbb{Z}_+$, entering the system with state $\hat{x}(t)$.

Remark 1: Observe that for $\chi(0) = 0_n$, the preceding set coincides with the maximal Φ_0-invariant set, say $\mathcal{R}^*$, of all states $\tilde{x}$ of Eq. (12), which are reachable from 0_n under the effect of disturbances. Moreover, $\mathcal{R}_t^*(\chi(0)) \to \mathcal{R}^*$ as $t \to \infty$ independently of $\chi(0)$. $\mathcal{R}^*$ can be remarkably smaller than $\mathcal{R}^*(\chi(0))$. In such a case, performance improvements would result if the condition (14) could be replaced by the less conservative condition $\tilde{\chi} \supset \mathcal{R}^*$. However, the latter can be used only if the $\tilde{x}(t)$ has already reached the steady state of whenever $\chi(0) \subset \mathcal{R}^*$. □

All of the preceding discussion allows one to convert the problem to the one of synthesizing a full-state CG for the auxiliary system

$$\begin{aligned} \hat{x}(t+1) &= \Phi\hat{x}(t) + Gg(t) + G_\varepsilon\varepsilon(t), \qquad \hat{x}(0) = 0_n \\ \hat{y}(t) &= H_y\hat{x}(t) \\ \hat{c}(t) &= H_c\hat{x}(t) + Lg(t) + \tilde{L}_\varepsilon\varepsilon(t) \end{aligned} \tag{15}$$

with $\varepsilon(t) \in \mathcal{E}, \forall t \in \mathbb{Z}_+$, a persistent bounded disturbance

$$\varepsilon(t) := [\xi'(t) \quad \tilde{x}'(t) \quad d'(t)]' \in \mathcal{E}, \qquad \mathcal{E} := \Xi \times \tilde{\chi} \times \mathcal{D} \tag{16}$$

$$G_\varepsilon := [-K \quad -KH_z \quad 0], \qquad \tilde{L}_\varepsilon := [0 \quad H_c \quad \tilde{L}] \tag{17}$$

Then, the CG unit (10) reduces to

$$g(t) := \underline{g}(\hat{x}(t), r(t)) \tag{18}$$

with $\hat{c}(t) \in \mathcal{C}_0, \forall t \in \mathbb{Z}_+, \mathcal{C}_0 \subset \mathcal{C}$ being a suitable restriction of $\mathcal{C}$ given in Eq. (20). A CG design problem of the form (15–18) has been recently solved, in a more general context, in Ref. 11. Hereafter, for the sake of completeness, we present the key arguments and summarize the main results and properties of the solution.

By linearity, one is allowed to separate the effects of initial conditions and commands from those of disturbances, for example, $x(t) = \hat{x}_g(t) + \hat{x}_\varepsilon(t)$, where $\hat{x}_g$ is the disturbance-free component (depending only on the initial state and commands) whereas $\hat{x}_e$ depends only on disturbances (including the estimation error). The same can be done for vectors c and y. Then, denote the disturbance-free steady-state solutions of Eq. (15), for a constant command sequence $g(t) = w, \forall t \in \mathbb{Z}_+$, as follows:

$$\begin{aligned} \hat{x}_w &:= (I - \Phi)^{-1}Gw \\ \hat{y}_w &:= H_y\hat{x}_w = w \qquad \text{[(Offset-free property 2)]} \\ \hat{c}_w &:= H_c\hat{x}_w + Lw = H_{\text{cw}}w \end{aligned} \tag{19}$$

where $H_{\text{cw}} := H_c(I_n - \Phi)^{-1}G + L$. Consider next the following set recursions[12]:

$$\mathcal{C}_0 := \mathcal{C} \sim \tilde{L}_\varepsilon\mathcal{E}, \qquad \mathcal{C}_k := \mathcal{C}_{k-1} \sim H_c\Phi^{k-1}G_\varepsilon\mathcal{E} \tag{20}$$

$$\mathcal{C}_\infty := \bigcap_{k=0}^{\infty} \mathcal{C}_k \tag{21}$$

where, given subset $\mathcal{A}, \mathcal{B} \subset \mathbb{R}^n$, $\mathcal{A} \sim \mathcal{B}$ denotes the set $\mathcal{A} \sim \mathcal{B} := \{a \in \mathcal{A} : a + b \in \mathcal{A}, \forall b \in \mathcal{B}\}$. Notice that the problem is not solvable if $\mathcal{C}_\infty$ is empty. This usually happens when the disturbance bounds are too large in comparisons with the actuators' ranges. On the contrary, if $\mathcal{C}_\infty$ is nonempty, all $\mathcal{C}_k$s, $\forall k \in \mathbb{Z}_+$ are such. In this case, all $\mathcal{C}_k$s are convex and compact and satisfy the nesting condition $\mathcal{C}_k \subset \mathcal{C}_{k-1}$. It has been shown in Ref. 12 that the sets $\mathcal{C}_k$ are nonconservative restrictions of $\mathcal{C}$ such that $\hat{c}_g(t) \in \mathcal{C}_\infty, \forall t \in \mathbb{Z}_+$ implies $\hat{c}(t) \in \mathcal{C}_0, \forall t \in \mathbb{Z}_+$, and, in turn, $c(t) \in \mathcal{C}, \forall t \in \mathbb{Z}_+$. The conclusion is that fulfillment of constraints can be ensured by only considering the disturbance-free evolution of Eq. (15).

Next consider, for a small enough $\delta > 0$, the sets

$$\mathcal{C}^\delta := \mathcal{C}_\infty \sim \mathcal{B}_\delta, \qquad \mathcal{W}^\delta := \left\{w \in \mathbb{R}^m : \hat{c}_w \in \mathcal{C}^\delta\right\} \tag{22}$$

where $\mathcal{B}_\delta$ is the ball of radius δ centered at the origin. In particular, $\mathcal{W}^\delta$, which we assume nonempty, is closed and convex and represents the set of all commands whose corresponding equilibrium point $\hat{c}_w$ satisfies the constraints with margin δ. Again, $\mathcal{W}^\delta$ empty prevents the problem from being solvable.

Then, our approach in selecting at each time t the CG action $g(t)$ as in Eq. (18) will be that of restricting the choice among all vectors of a suitable $\hat{x}(t)$-depending subset of $\mathcal{W}^\delta$, each vector of which, if constantly applied as a command to the system from the time instant t onward, gives rise to system evolutions that do not produce constraint violations. The preceding restriction is required because the condition $g(t) \in \mathcal{W}^\delta$ only ensures constraint fulfillment in steady state, namely, $\hat{c}_{g(t)} \in \mathcal{C}_0$, but nothing can be said about the transient from $\hat{x}(t)$ to $\hat{x}_{g(t)}$. If many choices exist, the vector that best approximates $r(t)$ is selected. Such a command is applied, a new state is measured, and the procedure is repeated. From the preceding discussion it follows that, at next time $t+1$, the preceding choice $g(t)$ is still admissible, although not necessarily the best.

All preceding discussion substantiates in the following procedure. Consider the following family of constant virtual command sequences:

$$g_w(\cdot) = \left\{g(k) \equiv w \in \mathcal{W}^\delta, \quad \forall k \in \mathbb{Z}_+\right\} \tag{23}$$

along with the following quadratic selection index:

$$J(r(t), w) := \|w - r(t)\|^2_{\Psi w} \tag{24}$$

where $\|x\|^2_\Psi := x'\Psi x$, $\Psi_w = \Psi'_w > 0_m$. Moreover, define the set $\mathcal{V}(x)$ as

$$\mathcal{V}(x) = \left\{w \in \mathcal{W}^\delta : \hat{c}_g(k, x, g(\cdot)_w) \in \mathcal{C}_k, \quad \forall k \in \mathbb{Z}_+\right\} \tag{25}$$

where

$$\hat{c}_g(k, x, g(\cdot)_w) := H_c\left(\Phi^k x + \sum_{i=0}^{k-1}\Phi^{k-i-1}Gw\right) + Lw \tag{26}$$

has to be understood as the disturbance-free virtual $\hat{c}$ evolution at virtual time k of $\hat{c}$ from the initial condition x at virtual time $k = 0$ under the constant command sequence $g_w(\cdot)$. As a consequences, $\mathcal{V}(x) \subset \mathcal{W}_\delta$. Moreover, if nonempty, it represents the set of all constant virtual command sequences $g_w(\cdot)$, whose corresponding $\hat{c}$ evolutions starting from x at time 0 satisfy the constraints for all $k \in \mathbb{Z}_+$. In particular, $\hat{c}_g(k, x, g(\cdot)_w) \in \mathcal{C}_k, \forall k \in \mathbb{Z}_+$ implies $\hat{c}(k, x, g(\cdot)_w, \varepsilon(\cdot)) \subset \mathcal{C}_0, \forall k \in \mathbb{Z}_+$, where $\hat{c}(k, x, g(\cdot)_w, \varepsilon(\cdot))$ is the overall set-valued virtual $\hat{c}$ evolution of Eq. (15) at time k from state x at time 0 under the command and disturbance actions.

Assume temporarily that, for every $t \in \mathbb{Z}_+$, $\mathcal{V}(\hat{x}(t))$ is nonempty, closed, and convex. This implies that the following minimizer uniquely exists for every $t \in \mathbb{Z}_+$:

$$w(t) := \arg\min_{w \in \mathcal{V}(\hat{x}(t))} J(r(t), w) \tag{27}$$

Thus, we adopt the following CG selection rule:

$$g(t) = w(t) \tag{28}$$

and repeat the same procedure at next time $t+1$.

All results so far obtained and all known properties of the CG unit (28) are summarized in the following theorem.

Theorem 1: Let assumptions (2) and (5) be fulfilled, and let $\mathcal{W}^\delta$ be nonempty. Consider the system (15) with the CG command given by Eq. (28), and let $\mathcal{V}(\hat{x}(0))$ be nonempty.

1) The set $\mathcal{V}(\hat{x}(t))$ is nonempty at each time $t \in \mathbb{Z}_+$ along the state trajectories corresponding to the CG action (27).

2) The minimizer in Eq. (27) uniquely exists at each time $t \in Z_+$ and can be obtained by solving a convex constrained optimization problem.

3) The set $\mathcal{V}(x)$, $\forall x \in \mathbb{R}^n$ is finitely determined, namely, there exists an integer k_0 such that if $\hat{c}_g(k, x, g(\cdot)_w) \in \mathcal{C}_k, k \in \{0, 1, \ldots k_0\}$, then $\hat{c}_g(k, x, g(\cdot)_w) \subset \mathcal{C}_k \forall k \in \mathbb{Z}_+$. Such a constraint horizon k_0 can be determined offline. See Ref. (11) for details.

4) The constraints are fulfilled for all $t \in \mathbb{Z}_+$.

5) The sequence of CG actions $\{w(t)\}$ is bounded; in particular, whenever $r(t) \equiv r$, $g(t)$ monotonically converges in finite time either to r or to its best admissible approximation $\hat{w}_{\hat{r}}$,

$$g(t) \to \hat{w}_r := \arg \min_{w \in \mathcal{W}^\delta} \|w - r\|^2_{\Psi w} \tag{29}$$

Consequently, by the offset free condition (2),

$$\lim_{t \to +\infty} \hat{y}_g(t) = \hat{w}_r \tag{30}$$

where $\hat{y}_g$ is the disturbance-free component of $\hat{y}$.

Proof: It follows by using, mutatis mutandis, the same arguments used in Ref. 11. □

III. Application to a Bank-to-Turn Missile Steering Problem

Consider the following linear model of a bank-to-turn (BTT) missile pitch dynamic[13]:

$$a_z(t) = -0.6368 \frac{(s - 47.318)(s + 47.319)}{(s - 10.77)(s + 13.827)} \delta_e(t)$$

$$\dot{\theta}(t) = -701 \frac{s + 3.194}{(s - 10.77)(s + 13.827)} \delta_e(t)$$

where $a_z(t)$ is the normal acceleration, $\dot{\theta}(t)$ the pitch rate, and $\delta_e(t)$ the elevator angle that is allowed to take values in the following interval:

$$-4.2\,\text{deg} \le \delta_e(t) \le 4.2\,\text{deg} \tag{31}$$

The preceding model represents the short-period pitch dynamic for a BTT missile traveling at Mach 1.38 at an altitude of 10,000 ft. At those operation conditions the plant is unstable and nonminimum phase. In Ref. 13 was proposed the following linear control law:

$$\delta_e(t) = 4[(s+2)/s(s+35)](a_{\text{ref}}(t) - a_z(t)) + 2.2[1/(s+35)]\dot{\theta}(t) \tag{32}$$

in order to stabilize the pitch dynamic where $a_{\text{ref}}(t)$ stands for the desired normal acceleration profile to be tracked. Notice that, because the plant is unstable, closed-loop stability can be guaranteed via Eq. (32) in linear regimes only, when the constraints (31) are fulfilled.

We can observe that both $a_x(t)$ and $\dot{\theta}(t)$ are measurable and used for control. However, in order to make the problem harder we assume that only $a_z(t)$ is available to the CG and that it is corrupted by a bounded observation noise $\delta_0(t)$

$$-0.01 \le \delta_0(t) \le 0.01 \tag{33}$$

For a sampling period of $T = 0.005$ s, we find the following discrete-time closed-loop state-space representation:

$$\begin{aligned} x(t+1) &= \Phi x(t) + G a_{\text{ref}}(t) - G\delta_0(t), \qquad x(0) = 0 \\ y(t) &= a_z(t) = H_y x(t) \\ z(t) &= a_z(t) + \delta_0(t) = H_y x(t) + \delta_0(t) \\ c(t) &= \delta_e(t) = H_c x(t) + L a_{\text{ref}}(t) - L\delta_0(t) \end{aligned} \tag{34}$$

where

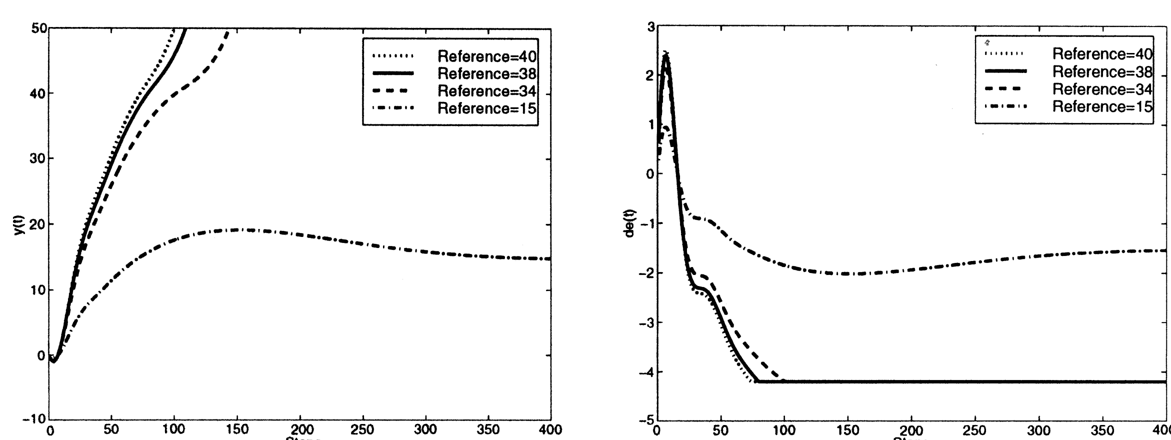
$$\Phi = \begin{bmatrix} 3.8203 & 1 & 0 & 0 \\ -5.4958 & 0 & 1 & 0 \\ 3.5301 & 0 & 0 & 1 \\ -0.8546 & 0 & 0 & 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0.017 \\ -0.0511 \\ 0.051 \\ -0.0169 \end{bmatrix} \tag{35}$$

$$H_y = [0 \quad 0 \quad 0 \quad 1], \qquad H_c = [1 \quad 0 \quad 0 \quad 0], \qquad L = 0.0093 \tag{36}$$

It is simple to verify that Eq. (34) satisfies the offset-free property in Eq. (2). Figure 2 reports the system responses to various steps reference signals without the use of the CG. From simulations it results that the presence of the input saturation produces a loss of stability for reference steps higher than $|a_{\text{ref}}| = 31.59$. In fact, notice in Fig. 2 that reference steps higher than $a_{\text{ref}} = 32$ produce elevator saturation and loss of stability.

In designing the CG unit, $\delta = 0.1$ was chosen. Correspondingly, the set $\mathcal{W}^{0.1}$ is an interval whose size depends on the observer's poles locations. Such a relationship is plotted in Fig. 3a, where it results that for maximizing the size of such a set it would be convenient to choose all observer's poles coinciding at nearly 0.93 and achieve $\mathcal{W}^{0.1} = [-42, 42]$. This would imply that all reference set points contained in the interval [−42, 42] can be tracked in steady-state without error. However, the corresponding index k_0 results excessively high (high online numerical burdens). The reasonable compromise of $\mathcal{W}^{0.1} = [-38, 38]$ and $k_0 = 5$ has been finally found by locating the observer's eigenvalues differently.

Figures 3 and 4 report the same experiment of Fig. 2 with the use of the CG. Notice how the CG device is able to handle reference set points of any size without any loss of stability at the expense of only a nonzero steady-state error for set points larger than $a_{\text{ref}} = 38$. This results especially from Fig. 4a, where the CG action $g(t)$ is plotted against time steps for different set points. In particular, the set point



a) Output $a_z(t)$ b) Constrained vector $\delta_e(t)$

Fig. 2 Closed-loop responses to various step reference signals a_{ref} without CG (stability for $a_{\text{ref}} \in [-31.59, 31.59]$.

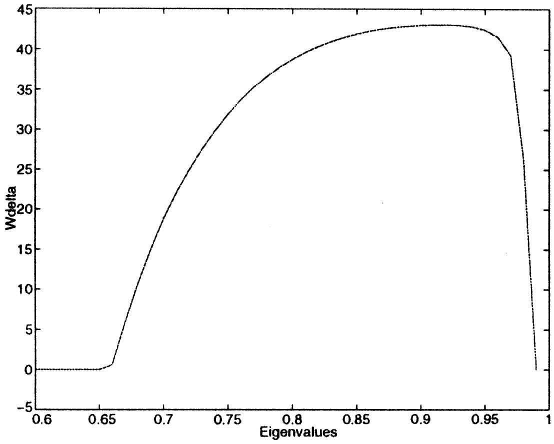


Fig. 3a $\mathcal{W}^{0.1}$ plotted against the position of the observer's eigenvalues (all coinciding).

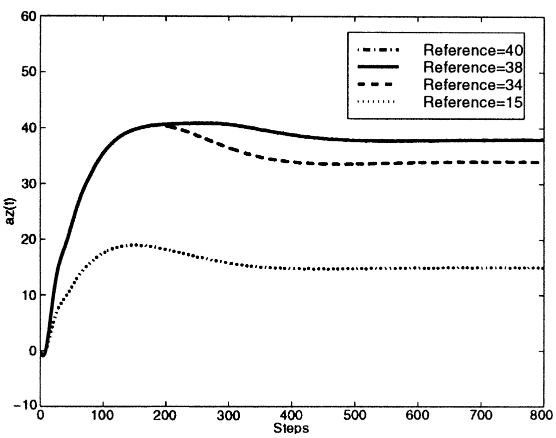


Fig. 3b Output $a_z(t)$.

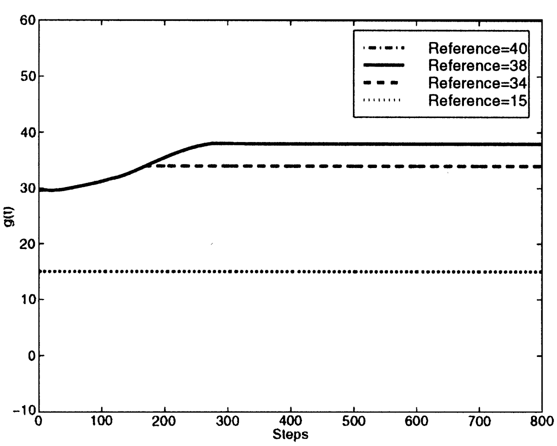


Fig. 4a CG action $g(t)$.

$a_{\text{ref}} = 15$ is not modified at all by the CG, $a_{\text{ref}} = 34$ is smoothed during the first 200 steps but has not steady-state error, whereas the CG actions corresponding to $a_{\text{ref}} = 40$ and 38 are essentially superimposed (the same in Figs. 3b and 4b). In fact, the set point $a_{\text{ref}} = 40$ cannot be achieved in steady state because it is out of the admissible range [−38, 38]. Notice also how the CG action converges in finite time to $w_r = 38$, namely, to the best steady-state admissible approximation of $a_{\text{ref}} = 40$.

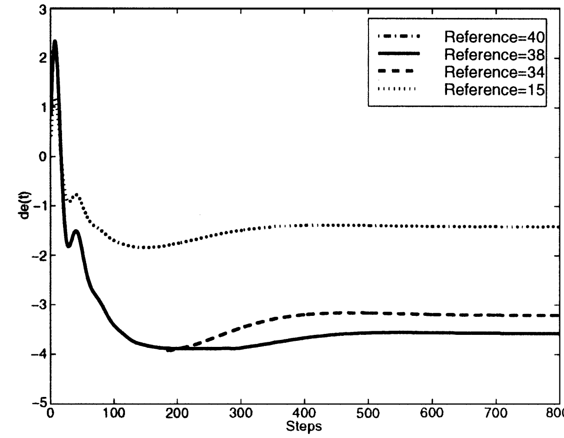


Fig. 4b Constrained variable $\delta_e(t)$.

IV. Conclusions

The Note has addressed the CG design problem in the case of partial state information. The problem is that of synthesizing a command input in such a way that a primal compensated control system can operate in a stable and linear way with satisfactory tracking performance and without constraint violations when the information on the state can only be estimated by noisy measurements.

The CG action calculation consists of solving online a finite dimension quadratic-programming optimization problem. An example of application to a bank-to-turn steering problem has been provided for supporting and clarifying the theory.

It has been shown that the proposed technique can ensure, over linear control methods, improvements on tracking performance in the presence of constraints up to an extent that justifies the modest increase of computing hardware required by the moderate computational burden of the CG algorithms.

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Adaptive Output Feedback Control Methodology: Theory and Practical Implementation Aspects

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Introduction

ADAPTIVE output feedback control using universal function approximators, such as artificial neural networks (NNs), is motivated by the multitude of practical applications for which accurate modeling is challenging or may even be impossible.[1–3] Recently, two direct output feedback approaches that do not rely on state estimation were presented in Refs. 4 and 5. These approaches incorporate similar approximate output feedback linearization, linear dynamic compensation of the ideally linearized model, and different adaptive NN-based elements to compensate for the model linearization errors. The design of the linear compensator may be challenging for systems with high relative degree r, requiring stabilization of r poles at the origin resulting from standard linearization.

The current work examines an extension to the approach in Ref. 4 by introducing a different feedback linearization scheme, useful for systems with high relative degree. Incorporating prior knowledge of the nonlinear system dynamics in the feedback linearization stage may greatly simplify the design of the linear compensator discussed earlier. It is particularly useful for applications in which part of the system dynamics, for example, actuator dynamics, are nearly linear with relatively well-known characteristics. Here we address several theoretical aspects that justify the procedure and that point out several practical design implications. Although presented in conjunction with the direct output feedback scheme of Ref. 4, the proposed linearization procedure can be used with any control design technique that relies on model inversion, for example, that of Ref. 5.

Problem Statement

Let the dynamics of a state observable nonlinear single-input/single-output (SISO) system be given in a state-space form by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u) \tag{1a}$$

$$y = h(\boldsymbol{x}) \tag{1b}$$

where $\boldsymbol{x} \in \boldsymbol{\Omega}_x \subset \Re^n$ is the state space vector of the system with possibly unknown dimension n, $u \in \Omega_u \subset \Re$ is the system input (control), $y \in \Omega_y \in \Re$ is its output (measurement) signal, and $\boldsymbol{f}(\cdot, \cdot)$, $h(\cdot)$ are unknown functions with appropriate input–output spaces.

Assumption 1. The dynamic system of Eqs. (1) satisfies the input–output feedback linearization conditions[6] with relative degree r.

Assumption 1 implies

$$y^{(i)} = \frac{\mathrm{d}h_{i-1}(\boldsymbol{x})}{\mathrm{d}t} = \frac{\partial h_{i-1}(\boldsymbol{x})}{\partial \boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x}, u) \triangleq h_i(\boldsymbol{x}) \tag{2}$$

for $i = 1, \ldots, r-1$ with $h_0(\boldsymbol{x}) = h(\boldsymbol{x})$ and

$$y^{(r)} = \frac{\mathrm{d}h_{r-1}(\boldsymbol{x})}{\mathrm{d}t} \triangleq h_r(\boldsymbol{x}, u) \tag{3}$$

It also implies that the system can be transformed into a normal form[6]

$$\dot{\boldsymbol{y}} = \boldsymbol{h}(\boldsymbol{y}, \boldsymbol{\xi}, u) \tag{4a}$$

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\chi}(\boldsymbol{y}, \boldsymbol{\xi}) \tag{4b}$$

where $\boldsymbol{y} \triangleq [y \;\; \dot{y} \;\; \cdots \;\; y^{(r-1)}]^T$ and $\boldsymbol{\xi} \triangleq [\xi_1 \;\; \xi_2 \;\; \cdots \;\; \xi_{n-r}]^T$.

The output feedback control law discussed in this work is designed so that the system output y tracks a bounded reference signal or trajectory $\boldsymbol{y}_{\mathrm{tr}} \in \Omega_{\mathrm{tr}} \subset \Re$, which is assumed to be r times differentiable with bounded derivatives. The reference trajectory and its derivatives are grouped into $\boldsymbol{y}_{\mathrm{tr}} = [y_{\mathrm{tr}} \;\; \dot{y}_{\mathrm{tr}} \;\; \cdots \;\; y_{\mathrm{tr}}^{(r-1)}]^T$. The tracking error and the tracking error vector are defined as

$$\tilde{y} = y_{\mathrm{tr}} - y \tag{5}$$

$$\tilde{\boldsymbol{y}} = \boldsymbol{y}_{\mathrm{tr}} - \boldsymbol{y} \tag{6}$$

Assumption 2. The system $\dot{\boldsymbol{\xi}} = \boldsymbol{\chi}(\boldsymbol{y}_{\mathrm{tr}}, \boldsymbol{\xi})$ has a unique steady-state solution $\bar{\boldsymbol{\xi}}$. Moreover, with $\tilde{\boldsymbol{\xi}} = \bar{\boldsymbol{\xi}} - \boldsymbol{\xi}$, the system

$$\dot{\tilde{\boldsymbol{\xi}}} = \boldsymbol{\chi}(\bar{\boldsymbol{\xi}}, \boldsymbol{y}_{\mathrm{tr}}) - \boldsymbol{\chi}(\bar{\boldsymbol{\xi}} - \tilde{\boldsymbol{\xi}}, \boldsymbol{y}_{\mathrm{tr}} - \tilde{\boldsymbol{y}}) \triangleq \tilde{\boldsymbol{\chi}}(\tilde{\boldsymbol{\xi}}, \tilde{\boldsymbol{y}}, \bar{\boldsymbol{\xi}}, \boldsymbol{y}_{\mathrm{tr}}) \tag{7}$$

has a continuously differentiable function $V_{\tilde{\xi}}(t, \tilde{\boldsymbol{\xi}})$ that satisfies

$$\eta_1 \|\tilde{\boldsymbol{\xi}}\|^2 \le V_{\tilde{\xi}}(t, \tilde{\boldsymbol{\xi}}) \le \eta_2 \|\tilde{\boldsymbol{\xi}}\|^2 \tag{8a}$$

$$\frac{\mathrm{d}V_{\tilde{\xi}}(t, \tilde{\boldsymbol{\xi}})}{\mathrm{d}t} \le -\eta_3 \|\tilde{\boldsymbol{\xi}}\|^2 + \eta_4 \|\tilde{\boldsymbol{\xi}}\| \|\tilde{\boldsymbol{y}}\| \tag{8b}$$

where $\eta_1, \eta_2, \eta_3 > 0$ and $\eta_4 \ge 0$ are independent of $\boldsymbol{y}_{\mathrm{tr}}$.

This assumption implies that the zero dynamics of the original system in Eqs. (1) are exponentially stable and the system is minimum phase.

Controller Design

The output feedback controller design is based on a nonstandard dynamic model linearization, linear dynamic compensation of the linearized system, and adaptive NN-based compensation of the model linearization error.

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